ANTICIPATION GUIDE

filled in

Individually read each statement and decide whether you agree or disagree. Check off the appropriate box in the "Before Reading" column. When prompted to do so, use your textbook and discuss each statement with your table partner(s). Has your opinion changed? Check off the appropriate box in the "After Reading" column. Use the additional space in the middle column to correct any statements you disagree with.

BEFORE READING		Use the additional space in this column to correct any statement you disagree with.	AFTER READING	
Agree	Disagree		Agree	Disagree
		A system of equations is a collection of two or more equations with ~~a different~~ the same set of unknowns. (7.1) 2 variables solution → pair of values that satisfy both equations		
		In solving a system of equations, we try to find values for each of the unknowns that will satisfy one of the equations in the system. (7.2, 7.4, 7.5) both		✓
		A system of two linear equations may have one solution, infinite solutions, or no solution. (7.6)	✓	
		Multiplying or dividing the equations in a linear system by a non-zero number, or ~~adding or subtracting~~ the ~~equations p~~roduces an equivalent system. (7.4) equivalent equation (does not change the graphs) solution that satisfies both equations		✓

equivalent system

(give their intersection points)

7.1 Developing Systems of Linear Equations (p.394)

A **linear system** is a set of two or more linear equations, each of which uses the **same** two variables. (Neither variable has an exponent - or each is to exponent 1). Each equation corresponds to a **straight line**.

Which of the following are linear systems?

$3x+5y=8$ $6x-3y=23$	$8x-2y-8=0$ $\frac{7}{6}x+3y=12$	$-4xy+y=67$ $6x-4y=12$	$x=3y+12$ $9x-2=5y$
✓	✓	X	✓

A solution to a linear system is one point (ie (x,y)) that satisfies both equations (ie LS = RS) – a common solution to both equations. This point must satisfy BOTH equations to be a solution for the **system.** When solving a linear system with two variables, we are asking where the **two lines will intersect**.

LS = RS for both.

Verify that the system below has a solution of x = 2, and y = 3

$x-y=-1$
$2x+3y=10$

LS: $2-3$ $=-1$ RS: -1

(2,3) is on this line.

LS: $2(2)+3(3)$ $4+9$ 13 LS: 10

(2,3) is not on this line.

(2,3) is not a solution to this system

Creating a System of Equations from a word problem

The ability to translate words of phrases into the language of mathematics is an important skill. One use of such skills is to create a system of linear equations.

When we develop or model a situation with linear equations, we must:

•Use the question to find out what we are looking for. We will use one variable for each unknown (two variables per equation). Often the unknowns we are looking for are at the end of the information provided. Write two "let" statements. (ie *let x = the number*). This is a short phrase to restate what you are being asked to find in a word problem. These statements will explain briefly what the 2 variables represent.

•Organize the information given in the question, devise a plan, and formulate (translate) TWO equations (each with two variables) to make the linear system

•Solve the system using one of three methods (graphing, substitution, elimination) to find the common solution (the intersection point of the two lines) – the solution of the system

•Substitute the intersection point into each equation to check the solution.

For 7.1, we will be focusing on 1st, 2nd, 4th bullets above

Example 1

2 equations
2 variables.

a) Create a linear system to model this situation:

The perimeter of a Nunavut flag is 16ft. Its length is 2ft. longer than its width.

l

w

$l+l+w+w=16$

$\boxed{2l+2w=16}$

$l=w+2$

let l = length (ft)
w = width

l w

b) Denise has determined that the Nunavut flag is 5ft. long and 3ft. wide. Use the linear system from part *a* to verify that Denise is correct.

LS	RS	LS	RS
$2(5)+2(3)$		5	$3+2$
$=10+6$			$=5$
$=16$	✓ 16		✓

L=5
W=3
is right.

<u>Example 2</u>

\$ value of cans + bottles

\# of cans + bottle
cans + bottles

a) Create a linear system to model this situation:

A student in Calgary raised \$195 by collecting 3000 items for recycling. She received 5 cents for each pop can and 20 cents for each large plastic bottle.

let C = # of pop cans
B = # of plastic bottles

(#) $C+B=3000$
(\$ amount) $0.05C+0.20B=195$

b) The school collected 2700 pop cans and 300 plastic bottles. Use the linear system to verify these numbers.

LS	RS	LS	RS
$2700+300$	3000	$0.05(2700)+0.20(300)$	195
3000	✓	$135+60$	
		195	✓

Try it:

ai) Chub and Smelt are each sold by the kilogram. Chub sells for \$2.50/kg and Smelt sells for \$1.50/kg. The total cost of one order of 16 kg of fish is \$30. Create a linear system to model this situation.

Let c = kg of chub
Let T = kg of smelt

(\$ amount) $2.50c + 1.50T = 30$
(#) $c + T = 16$

aii) Usethe linear system to verify 10 kg of smelt and 6 kg of chub purchased.

LS	RS	LS	RS
$= 2.50(6) + 1.50(10)$ $= 15 + 15$ $= 30$ ✓	30	$6+10$ $=16$ ✓	16

10kg smelt and 6 kg chub is a solution to this system

b) A store sells wheels for roller skates in packages of 4 and wheels for inline skates in packages of 8. Explain the meaning of each variable. Explain what each equation represents.

i # wheels in inline skates
r # wheel roller skate

$$8i + 4r = 440$$ # of wheels

$$i + r = 80$$ 80 packages total (sum of each kind of package)

<u>SYSTEM OF EQUATIONS CAN BE SOLVED BY:</u>

1) Graphing
2) Elimination Method (Addition/Subtraction)
3) Substitution Method

use intercepts
x-intercept $y=0$ y-intercept $x=0$
or
convert to $y = mx + b$

1) GRAPHING

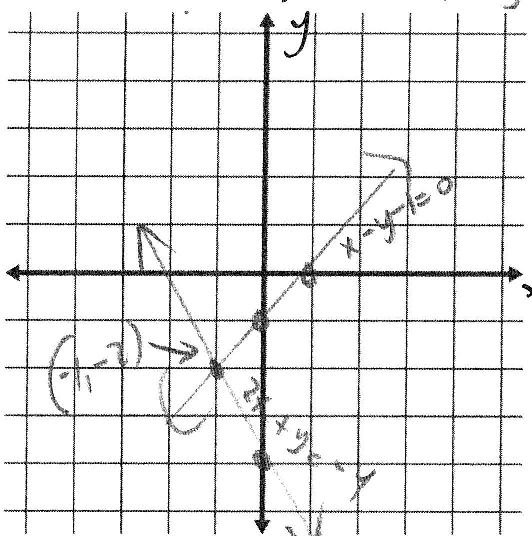
Ex. Solve the systems below by graphing. **Draw the lines using any method**

(The solution is the point where the two lines cross. Choose a scale and draw lines so that you can see the intersection on the cartesian plane. Use a pencil. You might have to erase and redraw.).

a)
$x - y - 1 = 0 \rightarrow y = x - 1$
$2x + y = -4 \rightarrow y = -2x - 4$

Solution: $(-1, -2)$

$LS = RS$ for both



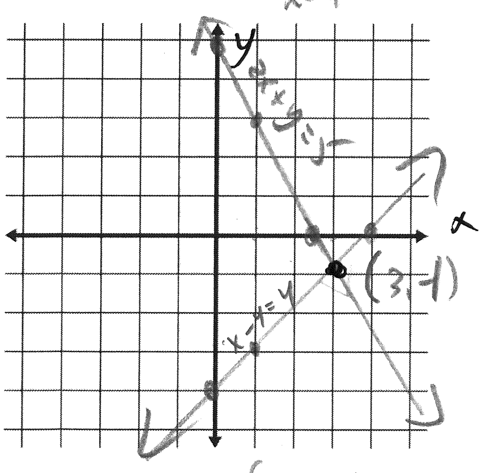
out of room instead of ↓2 → 1 can do ↑2 ← 1

check:
LS RS
$-1 + 2 - 1 = 0$ $\quad 0$ ✓

LS RS
$2(-1) - 2$ $\quad -4$
$-2 - 2$
-4 ✓

b)
$2x + y = 5 \rightarrow y = 5,\ x = \frac{5}{2}$ or $y = -2x + 5$ $\quad m = \frac{-2}{1}$ $(0, 5)$
$x - y = 4 \rightarrow y = -4,\ x = 4$ $\quad y = x - 4$

$-y = -x + 4$
$y = x - 4$



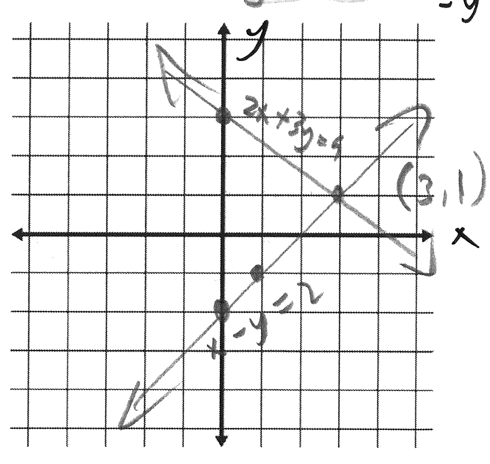
Solution: $(3, -1)$

check: RS LS
$2(3) - 1$ $\quad 5$
$6 - 1$
5 ✓

LS RS
$3 + 1$ $\quad 4$
4 ✓

c)
$2x + 3y = 9 \rightarrow 3y = -2x + 9$, $y = -\frac{2}{3}x + 3$
$x - y = 2 \rightarrow y = x - 2$

$-y = -x + 2$



Solution: $(3, 1)$

Can check by inspection (mental math)

THINK/PAIR/SHARE

Write out the appropriate steps to solve the following question. Use appropriate language. Explain each step with plenty of detail. Solve the problem algebraically first.

a) Write a linear system to model this situation:
To visit the Clear Lake interpretive centre in Wasagaming, Manitoba, the admission fee is \$5 for a student and \$9 for an adult. In one hour, 32 people entered the centre and a total of \$180 in admission fees was collected.

b) Graph the linear system then solve this problem: How many students and how many adults visited the centre during this time?

ALGEBRAIC SOLUTION	WRITTEN SOLUTION

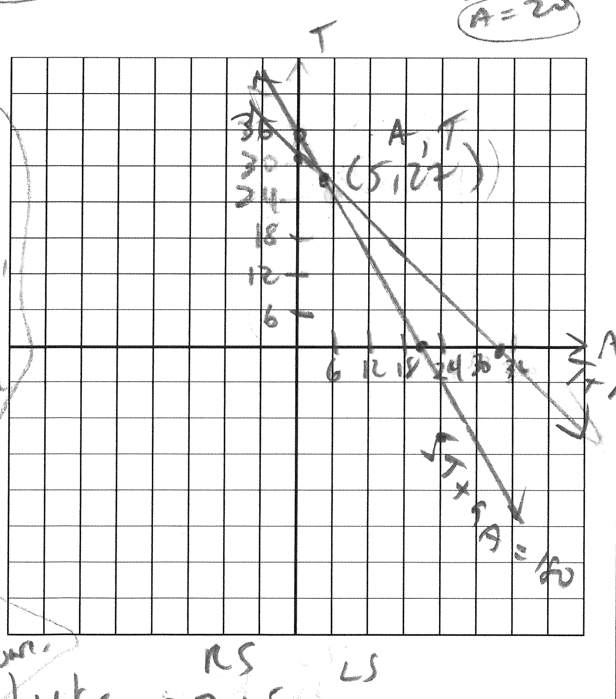
let T = # students
A = # adults

With big numbers, intercept method may be easier

$T + A = 32$
$5T + 9A = 180$

$T + 0 = 32$
$T = 32$
$0 + A = 32$
$A = 32$

$5T + 9(0) = 180$
$T = \frac{180}{5} = 36$ (0,36)
$5(0) + 9A = 180$
$9A = 180$
$\frac{9A}{9} = \frac{180}{9}$
$A = 20$ (20,0)



With a graph like this, answer may be approximate as lines don't cross on exact coordinates.

Check: RS 27+5 = 32, LS 32
$5(27) + 9(5) = 135 + 45 = 180$ ✓ LS 180

5 ADULTS 27 STUDENTS

① Write an expression about sum of number of each kind of person to equal 32.

② Write expression about sum of total paid by each group of people, to equal 180

③ Label each axis with variables. Graph each using intercept method. Label each line.

④ Write point of where lines cross.

⑤ Check point by substituting into LS RS for both equations. Must work for both.

P.409-410 #4, 6, 8, 19A

Solve the following system of equations:

1. $3x + 4y = -4$ ①
   $x + 2y = 2$ ②

② $x + 2y = 2$

$x = \boxed{-2y + 2}$

① $3(-2y+2) + 4y = -4$

$-6y + 6 + 4y = -4$

$-2y + 6 = -4$

$-2y = -10$

$y = 5$

② $x + 2y = 2$

$x + 2(5) = 2$

$x + 10 = 2$

$x = -8$

$(-8, 5)$ solution

Check

Check both variables in both equations

② LS | RS
$-8 + 2(5)$ | 2
$= -8 + 10$
$= 2$ ✓

LS | RS
$3(-8) + 4(5)$ | -4
$= -24 + 20$
$= -4$ ✓

Steps

① Number the equations.

② Choose equation that looks easiest to solve for 1 variable.

③ Choose easiest variable to solve for = SOLVE!

④ Substitute expression into other equation, for that variable.

⑤ Simplify and solve.

⑥ Substitute value into either equation to find other variable.

⑦ Write point.

⑧ Check LS RS

2. $5x - 3y = 18$ ①
$4x - 6y = 18$ ②

Step 1: Number each equation. **Solve one of the equations for either x = or y =.** (similar to solving for y = mx + b. Choose the equation that's easiest to solve in this way. Might involve moving terms from one side to another. Might involve multiplying or dividing all terms by the coefficient of the variable you're solving for. denominator You might have fractions.)

② $4x - 6y = 18$
$-6y = -4x + 18$
$y = \frac{2}{3}x - 3$

Step 2: Substitute the solution from step 1 into the ~~second~~ other equation.

① $5x - 3\left(\frac{2}{3}x - 3\right) = 18$

Step 3: Solve this new equation

$5x - 3\left(\frac{2}{3}x\right) - 3(-3) = 18$
$5x - 2x + 9 = 18$
$3x + 9 = 18$
$3x = 9$
$x = 3$

Step 4: Solve for the second variable

① $5(3) - 3y = 18$
$15 - 3y = 18$
$-3y = 3$
$y = -1$

The solution is: (3, -1)

Check it by substituting in your number for x and y into one of the equations. Compare left and right side to see if that point works -if RS = LS.

	LS	RS		LS	RS
①	$5(3) - 3(-1)$	18	②	$4(3) - 6(-1)$	18
	$= 15 + 3$			$= 12 + 6$	
	$= 18$ ✓			$= 18$ ✓	

Try it: p. 444 $\left(-\frac{23}{3}, \frac{-55}{24}\right)$

Solve the following system of equations:

1. ① $\frac{1}{2}x - \frac{4}{5}y = -2$

② $y = \boxed{\frac{1}{4}x - \frac{3}{8}}$

$$① \quad \frac{1}{2}x - \frac{4}{5}\left(\frac{1}{4}x - \frac{3}{8}\right) = -2$$

$$\frac{1}{2}x - \frac{4}{5}\left(\frac{1}{4}x\right) - \frac{4}{5}\left(-\frac{3}{8}\right) = -2$$

$$\frac{1}{2}x - \frac{1}{5}x + \frac{3}{10} = -2$$

$$\frac{5}{10}x - \frac{2}{10}x = -\frac{20}{10} - \frac{3}{10}$$

$$\left(\frac{10}{3}\right)\frac{3}{10}x = -\frac{23}{10}\left(\frac{10}{3}\right)$$

$$x = \frac{-23}{3}$$

or

$$\frac{1}{2}x - \frac{4}{5}\left(\frac{1}{4}x - \frac{3}{8}\right) = -2$$

$$10\left(\frac{1}{2}x\right) - 10\left(\frac{1}{5}x\right) + 10\left(\frac{3}{10}\right) = (-2)10$$

$$5x - 2x + 3 = -20$$

$$3x + 3 = -20$$

$$3x = -23$$

$$x = \frac{-23}{3}$$

or

$$10\left(\frac{1}{2}x\right) - 10\left(\frac{4}{5}\right)\left(\frac{1}{4}x - \frac{3}{8}\right) = (-2)10$$

$$5x - 8\left(\frac{1}{4}x - \frac{3}{8}\right) = -20$$

$$5x - 2x + 3 = -20$$

etc.

$$① \quad \frac{1}{2}\left(\frac{-23}{3}\right) - \frac{4}{5}y = -2$$

$$30\left(-\frac{23}{6}\right) - 30\left(\frac{4}{5}y\right) = (-2)(30)$$

$$-115 - 24y = -60$$

$$+115 \qquad\qquad +115$$

$$\frac{-24y}{-24} = \frac{55}{-24}$$

$$y = -\frac{55}{24}$$

$$\left(-\frac{23}{3}, -\frac{55}{24}\right)$$

$\left(\frac{1}{6}, \frac{-13}{8}\right)$

2. ① $\frac{1}{2}x+\frac{2}{3}y=-1$

② $y=\boxed{\frac{1}{4}x-\frac{5}{3}}$

see p. 422 for 2 methods

① $\frac{1}{2}x+\frac{2}{3}\left(\frac{1}{4}x-\frac{5}{3}\right)=-1$

$\frac{1}{2}x+\frac{2}{3}\left(\frac{1}{4}x\right)+\frac{2}{3}\left(-\frac{5}{3}\right)=-1$

$\frac{1}{2}x+\frac{1}{6}x-\frac{10}{9}=-1$

$\frac{3}{6}x+\frac{1}{6}x=-\frac{9}{9}+\frac{10}{9}$

$\left(\frac{6}{4}\right)\frac{4}{6}x=\frac{1}{9}\left(\frac{6}{4}\right)$

$x=\frac{2}{12}$

$x=\frac{1}{6}$

② $y=\frac{1}{4}\left(\frac{1}{6}\right)-\frac{5}{3}$

$y=\frac{1}{24}-\frac{40}{24}$

$y=\frac{-39}{24}$

$y=\frac{-13}{8}$

$\left(\frac{1}{6}, -\frac{13}{8}\right)$

P. 425 4a, 5c, 11, 17, 19a

Recall a system of equations can be solved by:

1)Graphing
2)Elimination Method (Addition/Subtraction)
3)Substitution Method

2) ELIMINATION METHOD to solve a System of Equations (7.5 p. 428)

Ex. Solve the following system:

1 $x+y=5$
2 $x-y=7$

*Add the two equations (Make sure x's and y's are lined up)

$2x=12$
$x=6$

into either equation

Substitute the solution for x to solve for y.

1 $6+y=5$
$y=-1$

check:

LS	RS	LS	RS
$6-1$	5	$6-(-1)$	7
5 ✓		$6+1$	
		7 ✓	

Solution: $(6,-1)$

Procedure:

i) Number the equations. Simplify the equations if necessary and arrange them in like term columns. (ax+by=c)

ii) Check if opposites exist in a column (eg. –2y and 2y).
If "yes", add equations.
If "no", multiply equation(s) to create a column of opposites, then add. Add (or subtract) a multiple of one equation to (or from) the other equation, in such a way that either the x -terms or the y -terms cancel out, when adding.

iii) Solve the resulting equation in one variable.

iv) Substitute the solution from iii into any equation containing the other variable and solve for the second variable.

v) Check your answer!

Ex. 1 ① $2x + 7y = 24$
② $3x - 2y = -4$

$\times 3$ ① $6x + 21y = 72$
$\times -2$ ② $-6x + 4y = 8$

$25y = 80$

$\frac{25y}{25} = \frac{80}{25}$

$y = \frac{80}{25} = \frac{16}{5}$

① $2x + 7\left(\frac{16}{5}\right) = 24$

$2x + \frac{112}{5} = 24$

$2x = \frac{120}{5} - \frac{112}{5}$

$\frac{2x}{2} = \frac{8}{5} \div 2$

$x = \frac{\cancel{8}^{4}}{5} \cdot \frac{1}{\cancel{2}}$

$x = \frac{4}{5}$

solution

$\boxed{\left(\frac{4}{5}, \frac{16}{5}\right)}$

check

LS | RS
$2\left(\frac{4}{5}\right) + 7\left(\frac{16}{5}\right)$ | 24

$= \frac{8}{5} + \frac{112}{5}$

$= \frac{120}{5}$

$= 24$ ✓

LS | RS
$3\left(\frac{4}{5}\right) - 2\left(\frac{16}{5}\right)$ | -4

$= \frac{12}{5} - \frac{32}{5}$

$= -\frac{20}{5}$

$= -4$ ✓

Ex. 2 ① $\frac{3}{4}x - y = 2$

② $\frac{1}{8}x + \frac{1}{4}y = 2$

Check LS · RS

① $\frac{3}{4}x - y = 2$

$\times 4$ ② $\frac{1}{2}x + y = 8$

$4\left(\frac{3}{4}x\right) + \left(\frac{1}{2}x\right)4 = (10)4$

$3x + 2x = 40$

$5x = 40$

$x = 8$

$\boxed{(8, 4)}$

① $\frac{3}{4}(8)^2 - y = 2$

$6 - y = 2$

$-y = -4$

$y = 4$

Practice
p. 5 in 'big review' booklet 1–15
p. 437 #8
7.5 in chapter review booklet

7.6 DETERMINE THE NUMBER OF SOLUTIONS OF EACH LINEAR SYSTEM (p. 444)

Write each equation in slope-intercept form. Identify the slope and y-intercept of each line.

a) $\boxed{\begin{array}{l} (1)\ x + y = 3 \\ (2)\ -2x - y = -2 \end{array}}$

(1) $x + y = 3$

$y = -x + 3$

slope : -1

y-intercept : 3

(2) $-2x - y = -2$

$-y = 2x - 2$

$y = -2x + 2$

slope : -2

y-intercept : 2

These lines have different slopes and different intercepts. These lines will cross at one point and will therefore have one solution.

b) $\boxed{\begin{array}{l} (1)\ \mathbf{4x + 6y = -10} \\ (2)\ \mathbf{-2x - 3y = 5} \end{array}}$

(1) $4x + 6y = -10$

$6y = -4x - 10$

$y = -\frac{4}{6}x - \frac{10}{6}$

$y = -\frac{2}{3}x - \frac{5}{3}$

slope : $-\frac{2}{3}$

y-intercept : $-\frac{5}{3}$

(2) $-2x - 3y = 5$

$-3y = 2x + 5$

$y = -\frac{2}{3}x - \frac{5}{3}$

slope : $-\frac{2}{3}$

y-intercept : $-\frac{5}{3}$

These lines have the Same slopes and y - intercepts. In fact, if you simplify the first line by dividing all terms by -2, you'll see it simplifies to the Same as the second line.

These lines have the same equation and therefore are "coincident" (one line is on top of the other line). These lines have infinite solutions. (They cross at every single point!)

c) $\boxed{\begin{array}{l} 2x - 4y = -1 \\ 3x - 6y = 2 \end{array}}$

$2x - 4y = -1$	$3x - 6y = 2$
$-4y = -2x - 1$	$-6y = -3x + 2$
$y = +\frac{2}{4}x + \frac{1}{4}$	$y = +\frac{3}{6}x - \frac{2}{6}$
$y = \frac{1}{2}x + \frac{1}{4}$	$y = +\frac{1}{2}x - \frac{1}{3}$
slope : $\frac{1}{2}$	slope : $\frac{1}{2}$
y-intercept : $\frac{1}{4}$	y-intercept : $-\frac{1}{3}$

These lines have the the same slope but different y - intercepts. Because the lines have the same slope, it means they are parallel (but not coincident – they are above/below each other because they've got different y-intercepts). This system therefore has no solution because parallel lines will never intersect (cross).

In summary…

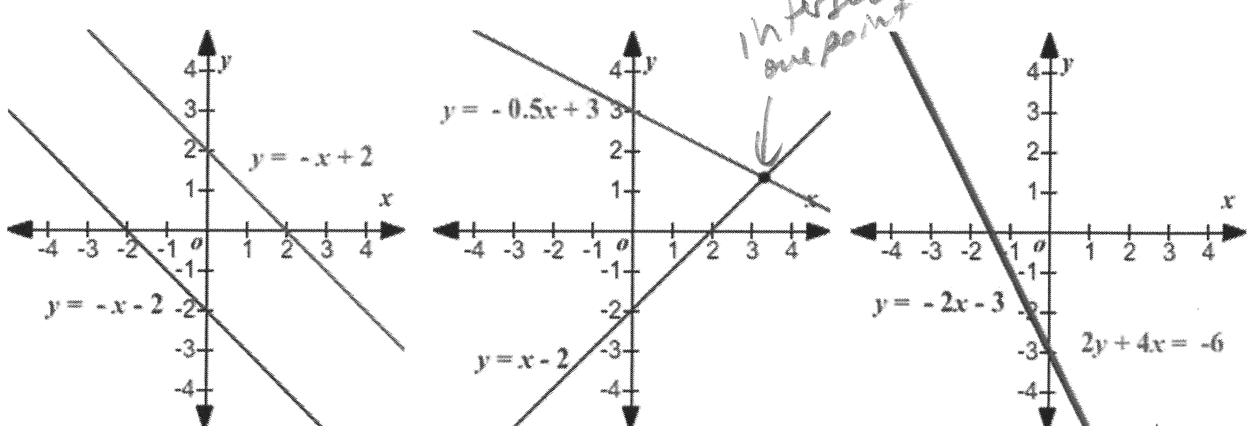
A **system of** linear equations is just a set of two or more linear equations.

In two variables (x and y)(x and y) , the graph of a system of two equations is a pair of lines in the plane.

There are three possibilities:

- The lines intersect at zero points. (The lines are parallel.)
- The lines intersect at exactly one point. (Most cases.) (different slope + y-intercept)
- The lines intersect at infinitely many points. (The two equations represent the same line.)

intersect at one point



parallel no intersection

coincident intersect at all points

Assignment – Systems of Equations

- **For full credit, show all work and place answers in the space provided ***

1. Given the following systems, determine if they have exactly **1 solution**, **no solution** or an **infinite number of solutions**:

a) ① $2x - 5y = 10$
② $10y - 4x = -20$

① $-5y = -2x + 10$
$y = \frac{2}{5}x - 2$

② $10y = 4x - 20$
$y = \frac{4}{10}x - 2$
$y = \frac{2}{5}x - 2$

coincident:
$m = \frac{2}{5}$ $b = -2$
same slope; same y-intercept

infinite solutions

b) ① $2y = 7x - 6$
② $5y = 7x + 15$

① $y = \frac{7}{2}x - 6$ $m = \frac{7}{2}$ $b = -6$
② $y = \frac{7}{5}x + 3$ $m = \frac{7}{5}$ $b = 3$

different slopes; different y-intercepts

One solution

c) ① $6x = 12y - 24$
② $30y - 15x + 30 = 0$

① $12y = 6x + 24$
$y = \frac{6}{12}x + 2$
$y = \frac{1}{2}x + 2$

$m = \frac{1}{2}$ $b = 2$

$m = \frac{1}{2}$ $b = -1$

② $30y = 15x - 30$
$y = \frac{15}{30}x - 1$
$y = \frac{1}{2}x - 1$

no solution

Same slopes; different y-intercepts
∴ parallel

2. Solve the following system by graphing.

LS | RS
$3(0)-2(-5)$ | 10
$0+10$
10 ✓

① $3x - 2y = 10$

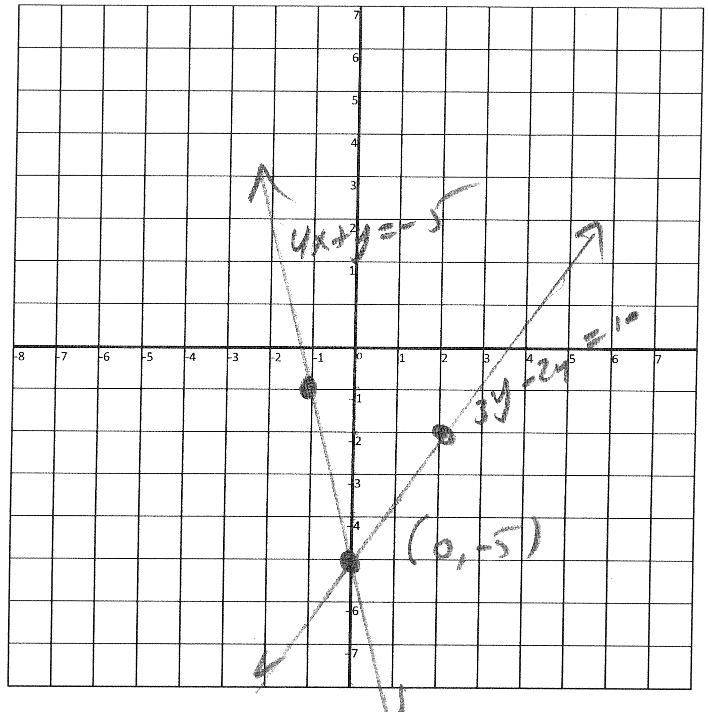
② $4x + y = -5$

LS | RS
$4(0)-5$ | -5
-5 ✓

① $-2y = -3x + 10$

$y = \frac{3}{2}x - 5$

② $y = -4x - 5$



3. Solve the following systems algebraically (solve one with elimination and the other with substitution):

(attach paper if needed)

a) ① $2x - 5y = -18$

② $8x - 13y = -58$

$\times -4$ ① $-8x + 20y = +72$

$8x - 13y = -58$

$7y = 14$

$y = 2$

① $2x - 5(2) = -18$

$2x = -8$

$x = -4$

$(-4, 2)$

LS | RS
$2(-4) - 5(2)$ | -18
$= -8 - 10$
$= -18$ ✓

LS | RS
$8(-4) - 13(2)$ | -58
$= -32 - 26$
$= -58$ ✓

LS | RS
$2\left(-\frac{80}{13}\right) + 4\left(\frac{79}{13}\right)$ | 12
$-\frac{160}{13} + \frac{316}{13}$
$= \frac{156}{13}$
$= 12$ ✓

LS | RS
$-\frac{80}{13} \div 5 + 3\left(\frac{79}{13}\right)$ | 17
$-\frac{80}{13} \cdot \frac{1}{5} + \frac{237}{13}$
$= -\frac{80}{65} + \frac{1185}{65}$
$= \frac{1105}{65}$
$= 17$ ✓

b) ① $2x + 4y = 12$

② $\frac{x}{5} + 3y = 17$

① $4y = -2x + 12$

$y = -\frac{2}{4}x + 3$

$y = -\frac{1}{2}x + 3$

② $\frac{x}{5} + 3\left(-\frac{1}{2}x + 3\right) = 17$

$\frac{x}{5} - \frac{3}{2}x + 9 = 17$

$\left(\frac{x}{5}\right) - \left(\frac{3}{2}x\right) = (8)10$

$2x - 15x = 80$

$-13x = 80$

$x = -\frac{80}{13}$

$\left(-\frac{80}{13}, \frac{79}{13}\right)$

① $2\left(-\frac{80}{13}\right) + 4y = 12$

$13\left(-\frac{160}{13}\right) + 13(4y) = (12)13$

$-160 + 52y = 156$

$52y = 316$

$y = \frac{316}{52} = \frac{79}{13}$